TURBULENT FRICTION AT TRANSONIC VELOCITIES

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Profiles of the velocity and tangential stress at the wall in a plane turbulent boundary layer were measured at transonic velocities in a broad range of Reynolds numbers.

1. The refinement of calculations of the turbulent frictional resistance on the basis of semiempirical methods is intimately related to obtaining detailed and reliable experimental data. The majority of experimental investigations have been devoted to the structure of the turbulent boundary layer and the frictional resistance at small subsonic velocities or at supersonic velocities. On the basis of these data, empirical dependences have been formulated for determining the boundary-layer characteristics and the frictional resistance; these are used over wider ranges of variation of the frictional parameters than in experimental conditions. In the early investigations, the frictional resistance was determined by indirect methods: from velocity profiles, from the readings of Preston tubes or Stanton attachments, from measurements of the heat-transfer characteristics using the Reynolds analogy, etc. [1, 2]. In recent years, there have been a series of investigations in which the frictional resistance in the supersonic and hypersonic velocity ranges was measured by a direct method, using floating gravimetric elements [3-5]. At present, in connection with the development of supercritical profiles and shoulders, there has arisen the need to determine the turbulent frictional resistance at transonic velocities and large Reynolds numbers. However, for this case, only a small number of experimental data are known, with the limited Reynolds numbers given in [6], and therefore further, more detailed investigations must be performed to provide the basis for semiempirical methods of calculating the frictional resistance at transonic velocities.

In the present work, the results of measuring the characteristics of a turbulent boundary layer over a broad range of Reynolds numbers at transonic flow velocities are reported. The frictional drag was measured using single-component floating gravimetric elements of magnetic-induction type, which give reliable and stable results [5]. The values of the coefficient c_f measured by the floating elements were found to be somewhat higher than those calculated from the semiempirical formula using the measured values of the momentum loss; the discrepancy is considerably reduced with increase in Reynolds number.

In recent years, sensors with a heated element or thermal film sensors have come to be widely used outside the Soviet Union [7]. The advantages of such sensors are that they are miniature, allowing measurements of the local frictional drag to be made practically at the point of the surface, that no marked perturbations are introduced in the flow, and that they are of low inertia, allowing measurements of the tangential-stress pulsations to be made in the presence of the appropriate apparatus for interpreting the signal. In the present work, the thermal film sensor was calibrated using a floating element, and the linearity of its

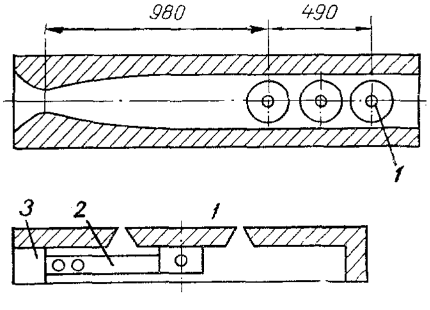

Fig. 1. Diagram of aerodynamic tube and construction of floating gravimetric element: 1) working disk of gravimetric element; 2) sensitive elastic plate; 3) induction coil. Dimensions in mm.

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theoretical characteristics was confirmed at transonic flow velocities over a broad range of Reynolds numbers.

2. Measurements of the characteristics of the turbulent boundary layer and of the frictional drag were made in a low-turbulence aerodynamic tube with a working-section length of 1 m and a cross section of 0.2×0.21 m. Detailed description of the construction and characteristics of the tube may be found in [5]. In the present investigation, a special nozzle with continuous walls was used, allowing the number M in the working region to be changed from 0 to 0.99 by varying the pressure in the prechamber and the ejectors. Changing the pressure in the prechamber from $0.2 \cdot 10^5$ to $8 \cdot 10^5$ bar allows the single Reynolds number $Re_1 = \rho_\infty U_\infty / \mu_\infty$ to be varied over a broad range (approximately from $3.5 \cdot 10^6$ to $1.4 \cdot 10^8$ m^{-1}).

Measurements were made at the side wall of the working section of the aerodynamic tube, at the centers of three windows at distances of 0.98, 1.225, and 1.47 m from the critical cross section of the nozzle (Fig. 1). Special investigations have shown that the boundary layer at the measurement points may be regarded as plane, to a good approximation. The frictional force was determined using a floating gravimetric element with a working-disk diameter of 8 mm (Fig. 1) and an ANCh-8 recorder. The velocity profiles in the boundary layer were measured using a total-pressure microattachment, which could be shifted across the boundary layer with a step of 0.01 mm. Detailed description of the construction and characteristics of these sensors and the other measuring equipment may be found in [5]. A thermocouple was fitted to determine the surface temperature T_w at the wall of the working section.

In addition, the possibility of using a thermal film sensor to measure the frictional resistance was investigated in the present work. The sensitive element of the sensor was a platinum film applied by the extrusion method onto the plane ends of a cylindrical quartz rod fitted flush with the surface in the wall of the working section. The film width with respect to the flux was 0.2 mm and along the stroke it was 1.0 mm; the film thickness was a few millimeters. The film-sensor reading was recorded by a standard thermoanemometric apparatus of the firm Diza Electroniks.

3. The method proposed by Clauser [8] for the analysis of the results of measuring the velocity profile in a plane turbulent boundary layer of incompressible liquid was to plot them graphically in the coordinates $[U/U_e, \log(yU_e/\nu)]$. If the linear section of the resulting curve, corresponding to the well-known wall law

$$\frac{U}{U_e} = \sqrt{\frac{c_f}{2}}\left[A \lg \frac{U_e y}{\nu} + C + A \lg \sqrt{\frac{c_f}{2}}\right], \tag{1}$$

where A and C are empirical constants equal to 5.5 and 5.6, respectively, is then isolated, the frictional coefficient c_f may be determined from the slope of the linear section. In Eq. (1), U is the longitudinal velocity at a distance y from the wall.

A generalization of the Clauser method to the case of the flow of compressible gas was proposed by Sivasegaram [9], writing the wall law for the turbulent boundary layer in the form

$$\sqrt{\frac{\rho_*}{\rho_e}}\,\frac{U}{U_e} = \sqrt{\frac{c_f}{2}}\left[A \lg \sqrt{\frac{\rho_*}{\rho_e}}\,\frac{\rho_e U_e y}{\mu_*} + C + A \lg \sqrt{\frac{c_f}{2}}\right], \tag{2}$$

where $\rho_* = (\rho_w + \rho)/2$; $\mu_* = (\mu_w + \mu)/2$.

Therefore, determining the coefficient c_f from the results of measuring the velocity profile in the boundary layer of compressible gas requires that they be plotted in the coordinates Y, X, where

$$Y = \sqrt{\frac{\rho_*}{\rho_e}}\,\frac{U}{U_e}; \quad X = \lg \sqrt{\frac{\rho_*}{\rho_e}}\,\frac{\rho_e U_e y}{\mu_*}.$$

To determine the velocity value from the readings of the total-pressure attachment, the value of the static temperature at the point of measurement must be known. For the temperature distribution across the boundary layer, the following formula is assumed to be valid

$$\frac{T}{T_e} = \frac{T_w}{T_e} + \left(1 - \frac{T_w}{T_e}\right)\left(\frac{U}{U_e}\right)^2, \tag{3}$$

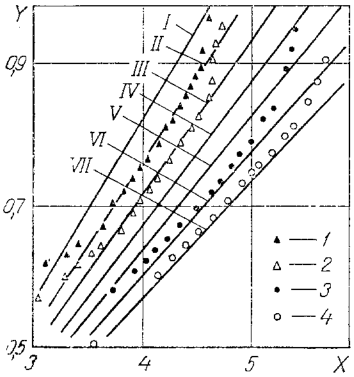

Fig. 2

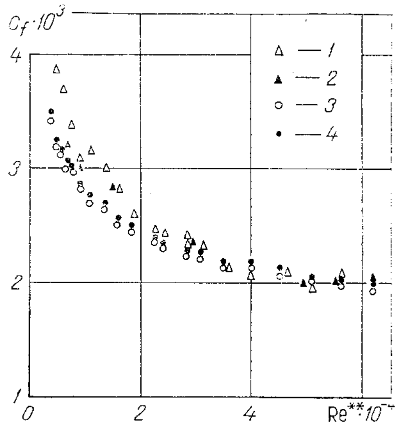

Fig. 3

Fig. 2. Velocity profiles in the boundary layer plotted in generalized Clauser coordinates: 1) $\text{Re}^{**} = 0.664 \cdot 10^4$, $M = 0.987$, $c_f = 2.75 \cdot 10^{-3}$; 2) $0.882 \cdot 10^4$, 0.960, $2.60 \cdot 10^{-3}$; 3) $2.82 \cdot 10^4$, 0.867, $2.05 \cdot 10^{-3}$; 4) $6.22 \cdot 10^4$, 0.986, $1.85 \cdot 10^{-3}$; I) $c_f = 3.0 \cdot 10^{-3}$; II) $2.8 \cdot 10^{-3}$; III) $2.6 \cdot 10^{-3}$; IV) $2.4 \cdot 10^{-3}$; V) $2.2 \cdot 10^{-3}$; VI) $2.0 \cdot 10^{-3}$; VII) $1.8 \cdot 10^{-3}$.

Fig. 3. Dependence of the local coefficient of surface friction on the Reynolds number: 1) measurement by means of a gravimetric floating element; 2) by the Clauser method; 3) by the Sommer—Short method; 4) by the Van Driest method.

where M_e is the value of M at the external boundary of the boundary layer. Some typical velocity profiles for the given investigation are shown in Fig. 2, together with a set of curves of Eq. (2) corresponding to different values of c_f, from which the value of the frictional coefficient may be determined for each of the measured velocity profiles.

Another indirect method of determining the local coefficient of frictional resistance is based on the relation between c_f and the momentum-loss thickness

$$\delta^{**} = \int_0^\infty \frac{\rho U}{\rho_e U_e} \left(1 - \frac{U}{U_e}\right) dy. \tag{4}$$

In the present work, use was made of the Karman—Shenkker formula, relating the local coefficient of frictional resistance in the incompressible flow c_{fi} to the Reynolds number Re_i^{**} [3]

$$c_{fi}^{-1} = 17{,}08 \lg^2 \text{Re}_i^{**} + 25{,}11 \lg \text{Re}_i^{**} + 6{,}012, \tag{5}$$

and of the transformation of compressible-flow parameters into incompressible-flow parameters

$$c_f = c_{fi}/F_c, \ \text{Re}^{**} = \text{Re}_i^{**}/F^{**}. \tag{6}$$

Two representations were used for the transformation coefficients F_c and F^{**}. According to the Sommer—Short method [3], the transformation coefficients are given by the formulas

$$F_c = T'/T_e; \ F^{**} = \mu_e/\mu', \tag{7}$$

in which the "determining" temperature T' is calculated from the relation

$$T' = T_e\left[1 + 0.035\,M_e^2 + 0.45\left(\frac{T_w}{T_e} - 1\right)\right]. \tag{8}$$

According to the Van Driest method [3]

$$F_c = \frac{r\,\frac{\gamma - 1}{2} M_e^2}{(\arcsin\alpha + \arcsin\beta)^2}, \quad F^{**} = \frac{\mu_e}{\mu_w}, \tag{9}$$

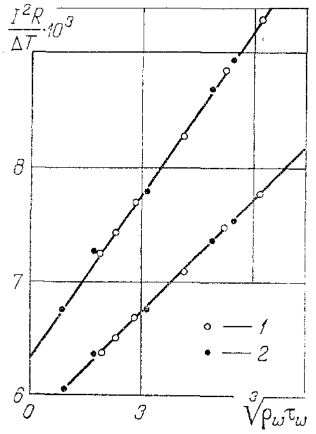

Fig. 4. Calibrational dependences of $I^2R/\Delta T$ (W/K) on $\sqrt[3]{\rho_w\tau_w}$ ($kg^{2/3}\cdot sec^{-2/3}\cdot m^{-1}$): 1) M = 0.95; 2) 1.48; ΔT = 139°K for the upper curve and 85°K for the lower curve.

$$\alpha=\frac{2A_0^2-B_0}{\sqrt{4A_0^2+B_0^2}},\ \beta=\frac{B_0}{\sqrt{4A_0^2+B_0^2}},\ A_0=\left(r\frac{\gamma-1}{2}M_e^2\frac{T_e}{T_w}\right)^{1/2},$$

$$B_0=\frac{T_e}{T_w}\left(1+r\frac{\gamma-1}{2}M_e^2-\frac{T_w}{T_e}\right). \tag{9}$$

Here r = 0.89 and $Re^{**}=\rho_e U_e\delta^{**}/\mu_e$.

The results of calculating the frictional coefficients c_f by the Clauser, Sommer—Short, and Van Driest methods are compared with the results of measuring c_f in the same conditions using a floating element in Fig. 3. On average, calculations from the measured values of δ^{**} using the Sommer—Short and Van Driest semiempirical methods are 5-6% lower than the results of direct measurement. Note also that these discrepancies diminish as the Reynolds number increases, and that the agreement is best between the values of c_f measured by the direct method and the values calculated by the Van Driest method. The values of c_f determined by the Clauser method are also in satisfactory agreement with the results of direct measurements.

4. The results of measuring c_f using a floating element were used to obtain a calibrational dependence of the thermal film sensor, with the aim of determining the possibility of its use in measuring friction at transonic velocities and large Reynolds numbers. According to the results of [10], with definite constraints on the film size, this dependence must take the form

$$\frac{I^2R}{\Delta T}=A_1+B_1\sqrt[3]{\rho_w\tau_w}, \tag{10}$$

where ΔT is the temperature difference between the sensor and the temperature of the flow around the wall; τ_w, tangential stress at the wall; A_1, B_1, constants independent of the Mach and Reynolds numbers. Equation (10) must be satisfied in the case when the thickness of the thermal boundary layer δ_T is less than the thickness of the viscous sublayer δ_v in the turbulent boundary layer. Estimates of these quantities from the well-known formulas

$$\delta_v^+=\frac{\delta_v\rho_e U_e}{\mu_e}\approx 5-10,\quad \delta_T\approx\frac{5x}{r\sqrt{Re_x}},$$

where x is the width of the heated film in the direction of the flow (x = 0.2 mm), show that this condition is always satisfied in the given measurements.

Thus, the aim of the calibration tests is to verify the linearity of the dependence of $I^2R/\Delta T$ on $\sqrt[3]{\rho_w\tau_w}$ and to determine the constants A_1 and B_1. The tests were performed at M = 0.95 and 1.48 (two different nozzles were used). Two arbitrarily chosen values ΔT = 85 and 139°K were investigated. The value of the parameter $\sqrt[3]{\rho_w\tau_w}$ was varied by changing the pressure in the prechamber of the tube, which led to change in the parameters of the turbulent boundary layer at the wall of the working section at the measurement point τ_w of the floating element and the thermal sensor. The results of the measurements are shown in Fig. 4. The spread of the data in Fig. 4 is no more than 3%, even though they correspond to different values of M and Re (i.e., τ_w). In view of the measurement accuracy, estimated at 7-10%, which is regarded as good for the turbulent friction, this value of the spread allows it to be supposed that the thermal sensor, having the advantage of small size and correspondingly small perturbation of the flow, will prove a valuable instrument for the investigation of frictional drag in a turbulent boundary layer over a broad range of Re and M, including transonic flow conditions.

Note also that the condition $\delta_V > \delta_T$ must also be observed for the given method. Thus, the calibrational dependence of $I^2R/\Delta T$ on $\sqrt[3]{\rho_W \tau_W}$ obtained in the present work for a film sensor with a width of 2 mm over the flow was found to be nonlinear. In this case, the given condition is violated.

NOTATION

c_f, local coefficient of frictional resistance; c_{fi}, local coefficient of frictional resistance for incompressible flow; I, current supplied to the heated element of the thermal film sensor; M, Mach number of the incoming flow; R, electrical resistance of the heated element of the thermal film sensor; r, coefficient of temperature recovery in the turbulent boundary layer; Re_1, unit R_e Reynolds number; Re**, Reynolds number calculated from the momentum-loss thickness; Re_i**, Reynolds number calculated from the momentum-loss thickness in incompressible flow; T, μ, ν, ρ, temperature, dynamic viscosity, kinematic viscosity, and density; T_e, ρ_e, temperature and density at the external boundary of the boundary layer; T_w, μ_w, ρ_w, temperature, viscosity, and density at the wall; T', μ', "determining" temperature according to Sommer—Short and viscosity calculated at this temperature; μ_∞, ρ_∞, viscosity and density of the incoming flow; μ_*, ρ_*, mean viscosity and density in the boundary layer; U, velocity; U_e, velocity of flow at the external boundary of the boundary layer; U_∞, velocity of incoming flow; x, y, longitudinal and transverse coordinates; γ, adiabatic coefficient for air; δ_V, thickness of viscous sublayer; δ_T, thickness of thermal boundary layer; δ**, momentum-loss thickness.

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